Thermodynamics of accelerating and rotating black holes

Muhammad Bilal and K. Saifullah

Department of Mathematics, Quaid-i-Azam University, Islamabad, Pakistan

Electronic address: `saifullah@qau.edu.pk`

Abstract: Thermodynamics of a large family of black holes from electrovacuum solutions of Einstein's equations is studied. This family includes rotating and non-accelerating black holes with NUT charge, and rotating and accelerating black holes. The surface gravity, Hawking temperature and the area laws for these black holes are presented. The first law of thermodynamics is also given. An interesting outcome of our analysis is the restriction obtained on the magnitude of acceleration for these black holes.

1. The Plebański-Demiański family of black holes

Plebański and Demiański [1] presented a large class of solutions of Einstein's equations with a possibly non-zero cosmological constant Λ, which includes, apart from other interesting solutions, the famous Kerr-Newman rotating black hole and hence the Kerr, the Reissner-Nordström, and the Schwarzschild black holes. This family includes, in particular, solutions for accelerating black holes also. The general form of the metric thus contains seven free parameters which characterize the mass m, electric and magnetic charges e and g respectively, Kerr-like rotation parameter a which is equal to angular momentum per unit mass i.e. $a = J/m$, the NUT (Newman-Unti-Tamburino) parameter l, acceleration of the source α and the cosmological constant Λ. We write the general Plebański-Demiański metric in the notation used in Refs. [2, 3] as

$$
\begin{aligned}
ds^2 = \frac{1}{\bar{\Omega}^2}\{&-\frac{Q}{\rho^2}[dt - (a\sin^2\theta + 4l\sin^2\frac{\theta}{2})d\phi]^2 + \frac{\rho^2}{Q}dr^2 + \frac{\rho^2}{P}d\theta^2 \\
&+\frac{P}{\rho^2}\sin^2\theta[adt - (r^2 + (a+l)^2)d\phi]^2\},
\end{aligned}
\tag{1.1}
$$

where

$$
\begin{aligned}
\bar{\Omega} &= 1 - \frac{\alpha}{\omega}(l + a\cos\theta)r, \\
\rho^2 &= r^2 + (l + a\cos\theta)^2, \\
P &= 1 - a_3\cos\theta - a_4\cos^2\theta, \\
Q &= (\omega^2 k + e^2 + g^2) - 2mr + \epsilon r^2 - 2\alpha\frac{n}{\omega}r^3 - (\alpha^2 k + \frac{\Lambda}{3})r^4, \\
a_3 &= 2\alpha\frac{a}{\omega}m \; - \; 4\alpha^2\frac{al}{\omega^2}(\omega^2 k + e^2 + g^2) - 4\frac{\Lambda}{3}al, \\
a_4 &= -\alpha^2\frac{a^2}{\omega^2}(\omega^2 k + e^2 + g^2) - \frac{\Lambda}{3}a^2, \\
\epsilon &= \frac{\omega^2 k}{a^2 - l^2} + 4\alpha\frac{l}{\omega}m - (a^2 + 3l^2)[\frac{\alpha^2}{\omega^2}(\omega^2 k + e^2 + g^2) + \frac{\Lambda}{3}], && (1.2) \\
n &= \frac{\omega^2 kl}{a^2 - l^2} - \alpha\frac{a^2 - l^2}{\omega}m + (a^2 - l^2)l[\frac{\alpha^2}{\omega^2}(\omega^2 k + e^2 + g^2) + \frac{\Lambda}{3}], && (1.3) \\
k &= [1 + 2\alpha\frac{l}{\omega}m - 3\alpha^2\frac{l^2}{\omega^2}(e^2 + g^2) - l^2\Lambda](\frac{\omega^2}{a^2 - l^2} + 3\alpha^2 l^2)^{-1}. && (1.4)
\end{aligned}
$$

Here ϵ and k are arbitrary real parameters, n is the Plebański-Demiański parameter and ω is the twist. If $\Lambda = 0$ or $\Lambda > 0$ then for the non-accelerating case the

metric will represent a single black hole, and for the accelerating case the metric will represent a pair of causally separated black holes which are accelerating away from each other in opposite directions. If $\Lambda < 0$ then the metric will represent a single black hole for small acceleration and a pair of black holes if the acceleration is large.

When $\alpha = 0 = l = g = \Lambda$ the line element reduces to the Kerr-Newman solution. Further, we get the Schwarzschild metric if the electric charge and rotation parameter vanish i.e. $e = 0 = a$. Therefore, the line element (1.1) is a very convenient metric representation of the complete class of accelerating, rotating and charged black holes. The metric is singularity free if $|a| < |l|$, and it has a Kerr-like ring singularity at $\rho = 0$ when $|a| \geq |l|$.

The rotating and accelerating black holes give rise to two types of horizons: the rotation horizons (analogous to the Kerr-Newman horizons) and two acceleration horizons. In this paper we study the thermodynamical properties of rotating black holes with NUT parameter and, rotating and accelerating black holes. The form of ergospheres for these black holes is given. We provide relations for their Hawking temperature and entropy. The first law of black hole thermodynamics is also discussed in the context of these spaces. As a result of our analysis we find an interesting relation which restricts the amount of acceleration these black holes can have.

2. Thermodynamics of the non-accelerating black holes

It can be seen from Eq. (1.1) that, when $\alpha = 0 = \Lambda$, we have $\omega^2 k = a^2 - l^2$ and hence $\epsilon = 1$, $n = l$, $P = 1$, and we get [3]

$$\begin{aligned} ds^2 = \frac{Q}{\rho^2}[dt - (a\sin^2\theta + 4l\sin^2\frac{\theta}{2})d\phi]^2 - \frac{\rho^2}{Q}dr^2 - \rho^2 d\theta^2 \\ -\frac{\sin^2\theta}{\rho^2}[adt - (r^2 + (l+a)^2)d\phi]^2, \end{aligned} \tag{2.1}$$

where

$$\rho^2 = r^2 + (l + a\cos\theta)^2, Q = (a^2 - l^2 + e^2 + g^2) - 2mr + r^2, \tag{2.2}$$

which is the Kerr-Newman-NUT solution. We note that $Q = 0$ gives the expression for locations of inner and outer horizons of the black hole as [3]

$$r_\pm = m \pm \sqrt{m^2 + l^2 - a^2 - e^2 - g^2}, \tag{2.3}$$

where $m^2 \geq a^2 + e^2 + g^2 - l^2$.

Here we discuss the formation of ergospheres in these black holes. We know that ergosphere are characterized by [4]

$$g_{tt} = 0. \tag{2.4}$$

So from Eqs. (2.1) and (2.2)

$$r^2 - 2mr - l^2 + e^2 + g^2 + a^2 \cos^2\theta = 0. \tag{2.5}$$

Its solution is

$$r_n(\theta) = m + \sqrt{m^2 + l^2 - e^2 - g^2 - a^2\cos^2\theta}, \tag{2.6}$$

which gives the ergosphere for the black hole. Now we see its relation with the outer horizon (2.3). For this we consider

$$0 \leq \cos^2\theta \leq 1, \tag{2.7}$$

which allows us to write

$$m^2 + l^2 - e^2 - g^2 - a^2 \leq m^2 + l^2 - e^2 - g^2 - a^2\cos^2\theta \leq m^2 + l^2 - e^2 - g^2, \tag{2.8}$$

$$\begin{aligned} & m + \sqrt{m^2 + l^2 - e^2 - g^2 - a^2} \\ \leq\ & m + \sqrt{m^2 + l^2 - e^2 - g^2 - a^2\cos^2\theta} \\ & \leq m + \sqrt{m^2 + l^2 - e^2 - g^2}, \end{aligned} \tag{2.9}$$

or

$$r_+ \leq r_n(\theta) \leq m + \sqrt{m^2 + l^2 - e^2 - g^2}, \tag{2.10}$$

$$r_+ \leq r_n(\theta) \leq r_a, \tag{2.11}$$

where r_a is the outer horizon of the corresponding Reissner-Nordström black hole with magnetic and NUT charges g and l respectively. The above relation has a beautiful information to interpret. Since the ergosphere is dependent on θ, so it will coincide with the outer horizon at $\theta = 0$ and stretches out for other values of θ. However, it cannot stretch beyond the outer horizon of the corresponding Reissner-Nordström black hole. At $\theta = \pi/2$, however, they coincide.

In order to discuss thermodynamics of these black holes with the NUT parameter, we observe that for a metric of the form

$$ds^2 = -F(r,\theta)dt^2 + \frac{1}{G(r,\theta)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{2.12}$$

the surface gravity, κ is given by [5]

$$\kappa = \frac{1}{\sqrt{-h}} \frac{\partial}{\partial x^a} \left(\sqrt{-h} h^{ab} \frac{\partial r}{\partial x^b} \right), \tag{2.13}$$

where h_{ab} is the second order diagonal metric made from $t-r$ sector of the metric and $h = \det h_{ab}$. Putting the values of h^{11} and $\sqrt{-h}$ from metric (2.1) we get

$$\kappa = \frac{1}{\sqrt{1 - \frac{a^2 \sin^2\theta}{Q}}} \frac{\partial f(r,\theta)}{\partial r}, \tag{2.14}$$

where

$$f(r,\theta) = \frac{Q}{\rho^2} \sqrt{1 - \frac{a^2 \sin^2\theta}{Q}}. \tag{2.15}$$

Thus Eq. (2.14) takes the form

$$\kappa = \frac{1}{\rho^2} \left(2(r-m) - \frac{2rQ}{\rho^2} + \frac{a^2 \sin^2\theta (r-m)}{(Q - a^2 \sin^2\theta)} \right). \tag{2.16}$$

At horizon $Q = 0$, $r \to r_+$ and using Eq. (2.3) at $\theta = 0$, this becomes

$$\kappa_h = \frac{(r_+ - m)}{\rho^2} = \frac{(r_+ - m)}{[r_+^2 + (l+a)^2]}. \tag{2.17}$$

or

$$\kappa_h = \frac{\sqrt{m^2 + l^2 - a^2 - e^2 - g^2}}{2m^2 + 2l^2 + 2al - e^2 - g^2 + 2m\sqrt{m^2 + l^2 - a^2 - e^2 - g^2}}, \tag{2.18}$$

It is important to note that if we put the NUT parameter $l = 0$ and the magnetic charge $g = 0$, then Eq. (2.18) reduces to the surface gravity for the Kerr-Newman black hole. Further if $e = 0 = a$ then the relation for the Schwarzschild black hole is obtained.

The relation (2.18) can also be written in terms of the inner and outer horizons by noting that

$$(r_+ - m) - (r_- - m) = 2(r_+ - m). \tag{2.19}$$

Thus Eq. (2.17) becomes

$$\kappa_h = \frac{r_+ - r_-}{2[r_+^2 + (l+a)^2]}. \tag{2.20}$$

We can also find the surface gravity by using the angular velocity [6]

$$\Omega = \frac{d\phi}{dt} = -\frac{g_{t\phi}}{g_{\phi\phi}}. \tag{2.21}$$

So we have

$$\Omega_h = \frac{a}{r_+^2 + (l+a)^2}. \tag{2.22}$$

or

$$\Omega_h = \frac{a}{2m^2 + 2l^2 + 2al - e^2 - g^2 + 2m\sqrt{m^2 + l^2 - a^2 - e^2 - g^2}}. \tag{2.23}$$

This is the angular velocity for the non-accelerating black hole. The formula for surface gravity in terms of angular velocity is [7]

$$\kappa_h = \frac{1}{2a}\Omega_h \left.\frac{dQ}{dr}\right|_{r=r_+}. \tag{2.24}$$

Using Eq. (2.23) and $dQ/dr = 2(r_+ - m)$, we get the same result as Eq. (2.18).

As we know that the temperature of a black hole is given by [4]

$$T = \frac{\kappa_h}{2\pi}. \tag{2.25}$$

Putting the value of κ_h from Eq. (2.18) we get

$$T = \frac{1}{2\pi}\left[\frac{\sqrt{m^2 + l^2 - a^2 - e^2 - g^2}}{2m^2 + 2l^2 + 2al - e^2 - g^2 + 2m\sqrt{m^2 + l^2 - a^2 - e^2 - g^2}}\right], \tag{2.26}$$

which is the temperature for the non-accelerating black holes. The temperature for the Kerr-Newman black hole can directly be deduced by putting $l = 0 = g$. Let us see the behavior of the temperature with the mass graphically.

We see from Figures 1 and 2 that if $l^2 > a^2 + e^2 + g^2$ then the temperature decreases with the increasing mass but will never be zero. If $l^2 < a^2 + e^2 + g^2$, the temperature will first increase when $m > \sqrt{a^2 + e^2 + g^2 - l^2}$ to the value of m where $dT/dm = 0$ and then it will decrease with the increasing mass. In this case temperature shows the same behavior as that of the Kerr-Newman black hole. We further note that for a fixed value of the NUT parameter, the maximum value of the temperature is inversely proportional to the collective magnitude of the rotation parameter and the electric and magnetic charges.

The horizon area of a rotating black hole is defined as [4]

$$A = \frac{4\pi a}{\Omega_h}. \tag{2.27}$$

Substituting from Eq. (2.23), this becomes

$$A = 4\pi\left[2(m^2 + l^2 + al + m\sqrt{m^2 + l^2 - a^2 - e^2 - g^2}) - e^2 - g^2\right]. \tag{2.28}$$

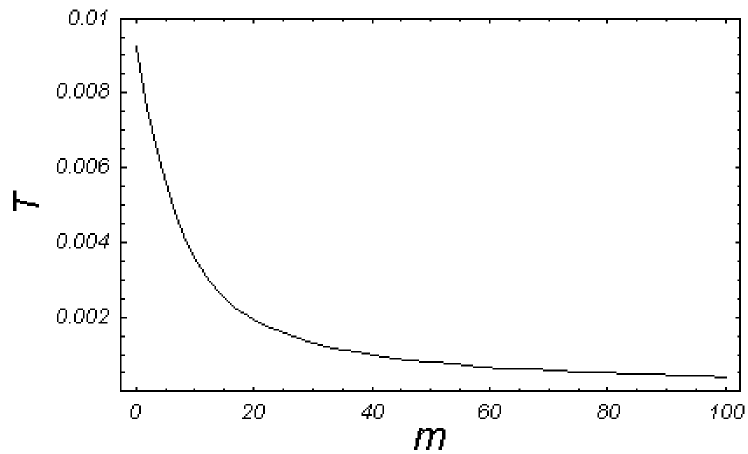

Figure 1: Temperature versus mass for the Kerr-Newman-NUT black hole when $l = 5, \alpha = e = g = 2$.

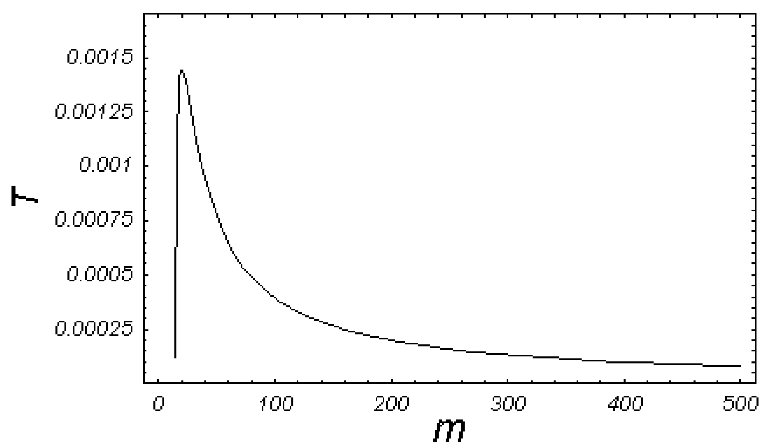

Figure 2: Temperature versus mass for the Kerr-Newman-NUT black hole when $l = 9, \alpha = e = g = 10$.

The entropy of a black hole [4]

$$S = \frac{A}{4}, \tag{2.29}$$

takes the form

$$S = \pi \left[2(m^2 + l^2 + al + m\sqrt{m^2 + l^2 - a^2 - e^2 - g^2}) - e^2 - g^2 \right]. \tag{2.30}$$

From Figure 3 we see that entropy is an increasing function of mass in accordance

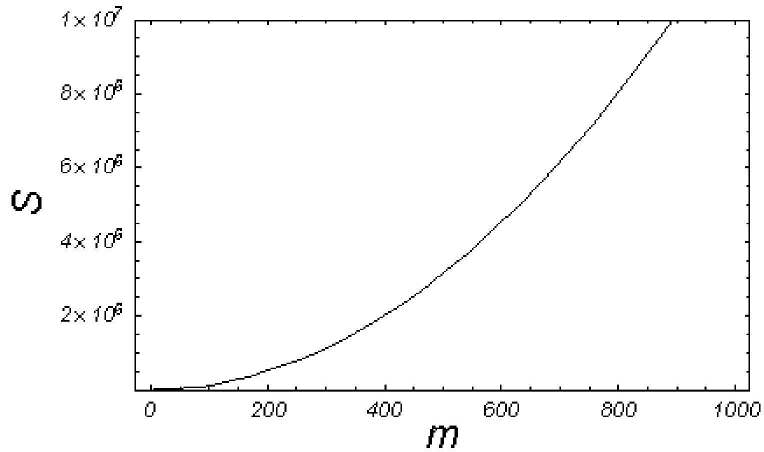

Figure 3: Entropy versus mass for the Kerr-Newman-NUT black hole when $l = 2, \alpha = e = g = 3$.

with the second law of black hole thermodynamics. The standard formula for the entropy of the Kerr-Newman black hole can be recovered from this.

Now, the first law of thermodynamics in the form of the law of conservation of mass is given by [4, 8]

$$dm = \frac{\kappa_h}{8\pi} dA + \Omega_h dJ + \Phi_h de, \tag{2.31}$$

where Φ_h is the electrostatic potential of the black hole [4]

$$\Phi_h = \frac{4\pi e r_+}{A}. \tag{2.32}$$

Putting the value of r_+ from Eq. (2.3) and of horizon area from Eq. (2.28), it becomes

$$\Phi_h = \frac{e[m + \sqrt{m^2 + \eta}]}{2(m^2 + l^2 + al + m\sqrt{m^2 + \eta}) - e^2 - g^2}, \tag{2.33}$$

where $\eta = l^2 - a^2 - e^2 - g^2$.

Substituting the values of κ_h from Eq. (2.18), Ω_h from Eq. (2.23) and Φ_h from above in Eq. (2.31), we get

$$dm = \frac{1}{\mu}\left[\frac{\sqrt{m^2 + \eta}}{8\pi} dA + a dJ + e(m + \sqrt{m^2 + \eta}) de\right], \tag{2.34}$$

where $\mu = 2m^2 + 2l^2 + 2al - e^2 - g^2 + 2m\sqrt{m^2 + \eta}$.

This is the first law of thermodynamics for the non-accelerating black holes.

3. Thermodynamics of accelerating and rotating black holes

As mentioned earlier the Plebański-Demiański metric includes the rotating and accelerating charged black holes. Now we consider another form of the metric which is free of NUT-like behavior i.e. we take $l = 0$. If we put $l = 0,\ k = 1$, in Eq. (1.1) then $\omega = a, a_3 = 2\alpha m,\ a_4 = -\alpha^2(a^2 + e^2 + g^2) - \frac{\Lambda}{3}a^2$ and substituting for ϵ and n, the line element (1.1) with $\Lambda \neq 0$ will take the form [9]

$$ds^2 = \frac{1}{\bar{\Omega}^2}\{-\frac{\bar{Q}}{\rho^2}[dt - a\sin^2\theta d\phi]^2 + \frac{P}{\rho^2}\sin^2\theta[adt - (r^2+a^2)d\phi]^2 + \frac{\rho^2}{\bar{Q}}dr^2 + \frac{\rho^2}{P}d\theta^2\}, \quad (3.1)$$

where

$$\bar{\Omega} = 1 - \alpha r\cos\theta, \rho^2 = r^2 + a^2\cos^2\theta, \quad (3.2)$$

$$P = 1 - 2\alpha m\cos\theta + \{\alpha^2(a^2 + e^2 + g^2) + \frac{\Lambda a^2}{3}\}\cos^2\theta, \quad (3.3)$$

$$\bar{Q} = \{(a^2 + e^2 + g^2) - 2mr + r^2\}(1 - \alpha^2 r^2) - \frac{\Lambda}{3}(r^2 + a^2)r^2. \quad (3.4)$$

The above metric contains six arbitrary parameters $m,\ e,\ g,\ \alpha,\ a$ and Λ which can be varied independently. We will not consider the cosmological constant in our discussion so we will put this equal to zero.

Here $\rho^2 = 0$ indicates the presence of a Kerr-like ring singularity at $r = 0$ and $\theta = \pi/2$. In this case $\bar{Q} = 0$ gives the expression for the locations of inner and outer horizons, which are identical to those of the non-accelerating Kerr-Newman black hole [9]

$$r_\pm = m \pm \sqrt{m^2 - a^2 - e^2 - g^2}, \quad (3.5)$$

where $a^2 + e^2 + g^2 \leq m^2$. However, in addition to these, there are acceleration horizons also at $r = 1/\alpha$ and $r = 1/\alpha\cos\theta$, which come from putting $\bar{Q} = 0$ and $\bar{\Omega} = 0$ respectively, and are coincident with each other at $\theta = 0$.

We first find the surface gravity of the accelerating and rotating black holes using Eq. (2.13). For $\Lambda = 0$, from Eq. (3.4) we have

$$\bar{Q} = (r^2 - 2mr + a^2 + e^2 + g^2)(1 - \alpha^2 r^2). \quad (3.6)$$

Thus Eq. (2.13) becomes

$$\kappa = \frac{\bar{\Omega}^2}{\rho^2}\left[\frac{1}{2}\frac{d\bar{Q}}{dr}\{1 + \frac{\bar{Q}}{\bar{Q} - Pa^2\sin^2\theta}\} - \frac{1}{\rho^2}2r\bar{Q}\right]. \quad (3.7)$$

At horizon, $\bar{Q} = 0$, therefore, Eq. (3.7) becomes

$$\kappa_h = \frac{\bar{\Omega}^2}{2\rho^2}\left[\frac{d\bar{Q}}{dr}\right]. \tag{3.8}$$

As $\bar{\Omega} \neq 0$ at $r = r_+$ we get

$$\kappa_h = \frac{\bar{\Omega}^2}{2\rho^2} 2[(r-m)(1-\alpha^2 r^2) - \alpha^2 r(r^2 - 2mr + a^2 + e^2 + g^2)]. \tag{3.9}$$

Since at outer horizon $r^2 - 2mr + a^2 + e^2 + g^2 = 0$, therefore, putting the value of ρ^2 and $\bar{\Omega}$ from Eq. (3.2) at $\theta = 0$, we get

$$\kappa_h = \frac{(r_+ - m)}{(r_+^2 + a^2)}(1 - \alpha r_+)^3(1 + \alpha r_+). \tag{3.10}$$

Using Eq. (3.5) this takes the form

$$\kappa_h = \frac{[1-\alpha(m+\sqrt{m^2-a^2-e^2-g^2})]^3[1+\alpha(m+\sqrt{m^2-a^2-e^2-g^2})]}{(\sqrt{m^2-a^2-e^2-g^2})^{-1}[2m^2-e^2-g^2+2m\sqrt{m^2-a^2-e^2-g^2}]}, \tag{3.11}$$

which is the surface gravity for the accelerating and rotating black holes at the outer horizon. Note that from Eq. (3.10) the surface gravity will vanish at the acceleration horizon, $r = 1/\alpha$ (and also for the other acceleration horizon at $\theta = 0$). In the above equation if $\alpha = 0$, it reduces to the relation of surface gravity for the non-accelerating case (Eq. (2.18)) at $l = 0$. Further if $g = 0$ the surface gravity for the Kerr-Newman black hole is obtained.

The relation (3.11) can also be written in terms of the inner and outer horizons. For this we use the relation $r_+ - r_- = (r_+ - m) - (r_- - m) = 2(r_+ - m)$ in Eq. (3.10) to get

$$\kappa_h = \frac{r_+ - r_-}{2(r_+^2 + a^2)}(1 - \alpha r_+)^3(1 + \alpha r_+). \tag{3.12}$$

Now, in this case the angular velocity from Eq. (2.21) becomes

$$\Omega = \frac{a[\bar{Q} - P(r^2 + a^2)]}{\bar{Q}a^2 \sin^2\theta - P(r^2 + a^2)^2}. \tag{3.13}$$

At horizon, $\bar{Q} = 0$, thus the angular velocity for the accelerating and rotating black holes can be written as

$$\Omega_h = \frac{a}{2m^2 - e^2 - g^2 + 2m\sqrt{m^2 - a^2 - e^2 - g^2}}. \tag{3.14}$$

Since angular velocity is calculated at the outer horizon which is not dependent on acceleration, therefore, this is the same as that of the non-accelerating case. Using this and Eq. (2.24) we get the same formula for the surface gravity as given in Eq. (3.11), and by $T = \kappa_h/2\pi$ we get the temperature for the accelerating and rotating black holes. We see that the factor $[1 - \alpha(m + \sqrt{m^2 - a^2 - e^2 - g^2})]$ can make the temperature negative for large values of α, so we have to allow only small magnitudes of acceleration. Note that the temperature vanishes at

$$1 - \alpha(m + \sqrt{m^2 - a^2 - e^2 - g^2}) = 0, \tag{3.15}$$

or

$$\alpha = \frac{1}{m + \sqrt{m^2 - a^2 - e^2 - g^2}}, \tag{3.16}$$

and in terms of mass we have

$$m = \frac{1 + \alpha^2(a^2 + e^2 + g^2)}{2\alpha}. \tag{3.17}$$

Thus to avoid the cases of extremal black holes and the violation of the third law of black hole thermodynamics, we need to restrict the values of α. We find that the permitted range is

$$\alpha < \frac{1}{m + \sqrt{m^2 - a^2 - e^2 - g^2}}. \tag{3.18}$$

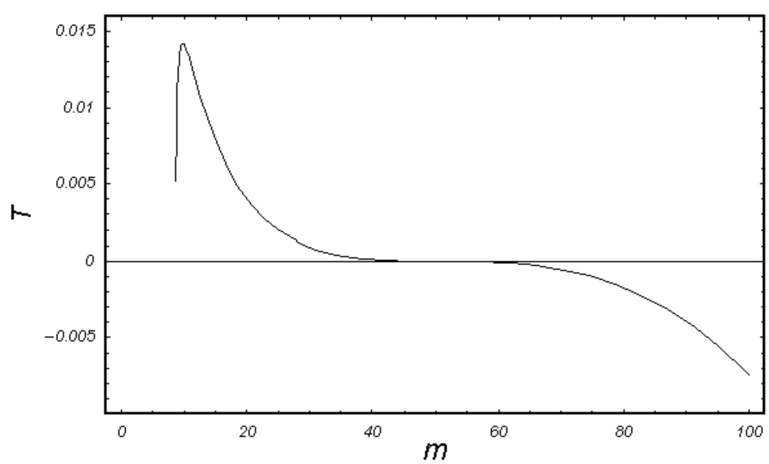

Figure 4: Temperature versus mass for the accelerating and rotating black hole when $\alpha = 0.01, a = e = g = 5$.

From Figure 4 we see that, for small values of α, the temperature of these black holes shows the same behavior as that of the non-accelerating black holes. When the

temperature becomes less than zero the black hole obviously ceases to exist. One can see that smaller is the acceleration greater is the maximum value of the temperature. Similarly, lesser the electric and magnetic charges and the rotation parameter greater is this maximum value.

In order to write the first law of thermodynamics, we note that the horizon area of the black hole using angular velocity from Eq. (3.14) is given by

$$A = 4\pi[2m^2 - e^2 - g^2 + 2m\sqrt{m^2 - a^2 - e^2 - g^2}]. \tag{3.19}$$

Using the values of r_+ and A from Eqs. (3.5) and (3.19) the electrostatic potential from Eq. (2.32) can be written as

$$\Phi_h = \frac{e[m + \sqrt{m^2 - \bar{\eta}}]}{[2m^2 - e^2 - g^2 + 2m\sqrt{m^2 - \bar{\eta}}]}, \tag{3.20}$$

where $\bar{\eta} = a^2 + e^2 + g^2$.

Now putting the values of κ_h, Ω_h and Φ_h from Eqs. (3.11), (3.14) and (3.20) in (2.31), we get the first law for the accelerating and rotating black holes as

$$\begin{aligned} dm = {} & \frac{1}{\bar{\mu}}[\frac{\sqrt{m^2 - \bar{\eta}}\ [1 - \alpha(m + \sqrt{m^2 - \bar{\eta}})]^3[1 + \alpha(m + \sqrt{m^2 - \bar{\eta}})]}{8\pi}dA \\ & +adJ + e(m + \sqrt{m^2 - \bar{\eta}})de], \end{aligned} \tag{3.21}$$

where $\bar{\mu} = 2m^2 - e^2 - g^2 + 2m\sqrt{m^2 - \bar{\eta}}$.

4. Conclusion

We have seen that the ergosphere for the Kerr-Newman-NUT black hole touches its outer horizon and stretches out upto a limit which corresponds to the horizon of the Reissner-Nordström black hole. The temperature for the Kerr-Newman-NUT black hole behaves like that of the Kerr-Newman black hole if the NUT parameter has less magnitude than that of the sum of the electric and magnetic charges along with the rotation parameter and shows a monotonically decreasing behavior when the NUT parameter has a magnitude more than the other three mentioned parameters. The maximum value of temperature is inversely proportional to the collective magnitude of the rotation parameter and the electric and magnetic charges.

For the accelerating and rotating case, we find that the large values of acceleration do not represent black holes. Only small values of acceleration are physically acceptable.

The entropy for both the accelerating and rotating, and the Kerr-Newman-NUT black holes justify the second law of thermodynamics. All the results and the thermodynamical quantities reduce to those of the Kerr-Newman black holes when $l = 0 = g = \alpha$. Further, they reduce to the Schwarzschild black holes in appropriate limits. It is worth mentioning here that the surface gravity and hence the temperature for accelerating and rotating black holes vanish on the acceleration horizon and violate the third law of black hole thermodynamics.

Acknowledgments

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